Relations and Functions

NCERT Class 12 Maths · Chapter 1
Exercise-wise notes · All NCERT questions solved · Important question types

$R \subseteq A \times A$ $f : A \to B$

Board weightage: Unit Relations and Functions = 8 marks (with Inverse Trigonometry). This chapter gives MCQs/assertion–reason and one 3–5 mark question: **prove a relation is an equivalence relation** or **prove a function is one-one / onto**.

This booklet follows the NCERT exercises: Exercise 1.1 (relations) → Exercise 1.2 (functions) → Miscellaneous.

How to study: learn the three tests (reflexive, symmetric, transitive) and the two tests (one-one, onto) as fixed steps – every question uses the same steps.

Colour	Meaning
Yellow highlight	Must remember
Blue card	Concept / rule + Note
Green box	Hinglish / □□□□□
Orange box	Results to remember
Cream box	Solved example – step by step
Red box	Exam tip

Start Here · Words You Must Know

Word	Meaning	Example / symbol
Relation	A subset of A × A: a rule that pairs elements of A. **Hinglish:** Set A ke elements ke beech ka **rishta** – ordered pairs (a, b) ka collection.	R ⊆ A × A, aRb
Empty relation	No element is related to any element. **Hinglish:** Koi bhi element kisi se related **nahi** – R = φ.	R = φ
Universal relation	Every element is related to every element. **Hinglish:** Har element **har element** se related – R = A × A.	R = A × A
Reflexive	Every element is related to itself. **Hinglish:** Har element **khud se** related: (a, a) sab ke liye.	(a, a) ∈ R for all a
Symmetric	If a is related to b, then b is related to a. **Hinglish:** a ~ b hai to **b ~ a bhi** hona chahiye – ulta bhi sahi.	(a, b) ∈ R ⇒ (b, a) ∈ R
Transitive	If a ~ b and b ~ c, then a ~ c. **Hinglish:** a ~ b aur b ~ c to **a ~ c** – rishta aage pass hota hai.	(a, b), (b, c) ∈ R ⇒ (a, c) ∈ R
Equivalence relation	Reflexive + symmetric + transitive, all three. **Hinglish:** Reflexive + symmetric + transitive – **teeno ek saath**.	“is equal to”, “is parallel to”

Word	Meaning	Example / symbol
Equivalence class	All elements related to a given element a. **Hinglish:** Ek element a se related **saare elements ka group** = [a].	[a] = {x : (x, a) ∈ R}
Function	Every element of A has exactly one image in B. **Hinglish:** A ke har element ka B mein **sirf ek** image.	f : A → B
One-one (injective)	Different inputs give different outputs. **Hinglish:** Alag input → **alag output**. Do input ka same output nahi.	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
Onto (surjective)	Every element of B is an image (range = codomain). **Hinglish:** B ka **har element** kisi na kisi ka image ho – range = codomain.	for every y ∃ x with f(x) = y
Bijective	Both one-one and onto. **Hinglish:** One-one bhi aur onto bhi – **perfect jodi**.	f(x) = 3 − 4x on R
Many-one	Two different inputs share one output (not one-one). **Hinglish:** Do alag input ka **same output** – jaise x^2 mein 2 aur −2.	x^2 : 2 and −2 → 4

Results to Remember

TESTS AND COUNTS

Quantity	Formula	Unit
Reflexive	$(a,a)\in R \quad \forall\, a\in A$	–
Symmetric	$(a,b)\in R \Rightarrow (b,a)\in R$	–
Transitive	$(a,b),(b,c)\in R \Rightarrow (a,c)\in R$	–
One-one	$f(x_1)=f(x_2) \Rightarrow x_1=x_2$	–
Onto	$\forall\, y\in B\ \exists\, x\in A:\ f(x)=y$	–
Relations on A, |A| = n	2^{n^2}	–
Functions A → B	$\|B\|^{\|A\|}$	–
One-one A → B (m ≤ n)	${}^{n}P_{m}=\frac{n!}{(n-m)!}$	–
Bijections A → A, |A| = n	$n!$	–
Finite set A → A	$\text{one}-\text{one} \Leftrightarrow \text{onto}$	–

Hinglish mein samjho:
Teen relation test aur do function test – yahi poora chapter hai. Counting: relations 2^{n^2}, bijections n!.

Exercise 1.1 · Types of Relations

Relation, empty and universal

$$R \subseteq A \times A, \qquad \phi \quad \text{and} \quad A \times A$$

✎ **Note:** A relation R on A is a set of ordered pairs (a, b) with a, b ∈ A. Write (a, b) ∈ R or aRb. Empty relation = no pair; universal relation = all pairs A × A. Both are called trivial relations.

Hinglish mein samjho:
Relation = A × A ka subset. Khali (φ) aur poora (A × A) – dono ko trivial relation kehte hain.

Reflexive, symmetric, transitive

$$\text{Reflexive}: \ (a, a) \in R \ \forall a$$

$$\text{Symmetric}: \ (a, b) \in R \Rightarrow (b, a) \in R$$

$$\text{Transitive}: \ (a, b), (b, c) \in R \Rightarrow (a, c) \in R$$

✎ **Note:** Check the three properties one by one. To prove a property, take general elements (a, b, c). To disprove it, one counter-example is enough – give actual numbers.

Hinglish mein samjho:
Teen test alag-alag karo. **Prove** karne ke liye general a, b, c lo; **galat** dikhane ke liye ek number wala example kaafi hai.

Equivalence relation and classes

$$[a] = \{x \in A : \ (x, a) \in R\}$$

✎ **Note:** A relation that is reflexive, symmetric AND transitive is an equivalence relation. It splits A into disjoint equivalence classes: every element is in exactly one class.

Hinglish mein samjho:
Teeno test pass = equivalence. Ye set ko **alag-alag groups** (classes) mein baant deta hai – har element ek hi group mein.

Hinglish mein samjho:
Equivalence prove karna = 3 steps: reflexive (a ~ a), symmetric (ulta), transitive (chain). Har step mein general letters use karo.

SOLVED EXAMPLE – Prove an equivalence relation (Ex 1.1 type)

Step 1: R on Z: (a, b) ∈ R if a − b is divisible by 3. Reflexive: a − a = 0, and 0 is divisible by 3.

$$(a, a) \in R \quad \forall a \in \mathbf{Z}$$

Step 2: Symmetric: if 3 divides a − b, then 3 divides −(a − b) = b − a.

$$a - b = 3k \ \Rightarrow \ b - a = 3(-k)$$

Why: −k is also an integer.

Step 3: Transitive: add the two differences.

$$a - b = 3k, \ \ b - c = 3m \ \Rightarrow \ a - c = 3(k + m)$$

Step 4: All three hold:

$$R \text{ is an equivalence relation}$$

SOLVED EXAMPLE – Counter-example (Ex 1.1 type)

Step 1: R on R: (a, b) ∈ R if a ≤ b². Reflexive? Try a = ½:

$$\frac{1}{2} \leq \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \text{is false}$$

Step 2: So (½, ½) ∉ R:

$$R \text{ is not reflexive}$$

Why: one counter-example is enough.

Exam tip: To show a relation is NOT reflexive/symmetric/transitive, write **one actual counter-example with numbers**. Writing only "not symmetric" gets no marks.

Exercise 1.1 · All NCERT Questions Solved

16 questions. Board favourites: **prove equivalence (Q8, Q9, Q11–Q14)** and **reflexive/symmetric/transitive checks (Q1, Q2, Q5)**. Q15–Q16 are MCQs.

NCERT – Q1 (i)

Type: Check the three properties

Question: Determine whether the relation is reflexive, symmetric and transitive: R in A = {1, 2, …, 13, 14}, R = {(x, y) : 3x − y = 0}.

Solution:

Step 1: List R: y = 3x with x, y in A.

$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

Step 2: Reflexive? (1, 1) ∉ R because 3(1) − 1 ≠ 0.

$$\text{not reflexive}$$

Step 3: Symmetric? (1, 3) ∈ R but (3, 1) ∉ R.

$$\text{not symmetric}$$

Step 4: Transitive? (1, 3), (3, 9) ∈ R but (1, 9) ∉ R.

$$\text{not transitive}$$

Answer: neither reflexive, nor symmetric, nor transitive

NCERT – Q1 (ii)

Type: Check the three properties

Question: R in N defined as R = {(x, y) : y = x + 5 and x < 4}.

Solution:

Step 1: List R:

$$R = \{(1, 6), (2, 7), (3, 8)\}$$

Step 2: (1, 1) ∉ R; (1, 6) ∈ R but (6, 1) ∉ R; there is no chain (a, b), (b, c) at all.

Answer: not reflexive, not symmetric, transitive (no counter-example possible)

NCERT – Q1 (iii)

★ **IMPORTANT** · **Type:** Check the three properties

Question: R in A = {1, 2, 3, 4, 5, 6}, R = {(x, y) : y is divisible by x}.

Solution:

Step 1: Reflexive: every x divides itself.

$$(x, x) \in R$$

Step 2: Symmetric? 1 divides 2, but 2 does not divide 1:

$$(1, 2) \in R,\ \ (2, 1) \notin R$$

Step 3: Transitive: if x | y and y | z then x | z.

$$y = kx,\ \ z = my \ \Rightarrow\ \ z = (km)x$$

Answer: reflexive and transitive, not symmetric

NCERT – Q1 (iv)

Type: Check the three properties

Question: R in Z, R = {(x, y) : x − y is an integer}.

Solution:

Step 1: x − x = 0 is an integer: reflexive.

Step 2: If x − y is an integer, so is y − x = −(x − y): symmetric.

Step 3: (x − y) + (y − z) = x − z is an integer: transitive.

Answer: reflexive, symmetric and transitive (equivalence)

NCERT – Q1 (v)

Type: Real-life relations

Question: R in the set of human beings at a fixed time: (a) x and y work at the same place, (b) live in the same locality, (c) x is exactly 7 cm taller than y, (d) x is wife of y, (e) x is father of y.

Solution:

Step 1: (a) and (b): “same place / same locality” – everyone is like themselves, it works both ways, and it passes along.

Step 2: (c) x is not 7 cm taller than x; if x is 7 cm taller than y, y is shorter; x 7 cm > y, y 7 cm > z gives x 14 cm > z.

Step 3: (d) x is not her own wife; if x is wife of y, y is husband; x wife of y and y wife of z is impossible, so it is vacuously transitive.

Step 4: (e) nobody is their own father; father of y is not y's son; father of father is grandfather.

Answer: (a), (b): equivalence; (c): none of the three; (d): only transitive; (e): none of the three

NCERT – Q2

★ **IMPORTANT** · **Type:** Counter-examples

Question: Show that R in R defined as R = {(a, b) : a ≤ b²} is neither reflexive nor symmetric nor transitive.

Solution:

Step 1: Reflexive fails for a = ½:

$$\frac{1}{2} \leq \frac{1}{4} \text{ is false}$$

Step 2: Symmetric fails for (1, 4):

$$1 \leq 16 \text{ true}, \quad 4 \leq 1 \text{ false}$$

Step 3: Transitive fails for (3, 2) and (2, 1.5):

$$3 \leq 4, \quad 2 \leq 2.25, \quad \text{but} \quad 3 \leq 2.25 \text{ false}$$

Answer: not reflexive, not symmetric, not transitive

NCERT – Q3

Type: Check the three properties

Question: Check whether R on {1, 2, 3, 4, 5, 6}, R = {(a, b) : b = a + 1}, is reflexive, symmetric or transitive.

Solution:

Step 1: R = {(1,2), (2,3), (3,4), (4,5), (5,6)}.

Step 2: (1, 1) ∉ R; (1, 2) ∈ R but (2, 1) ∉ R; (1, 2), (2, 3) ∈ R but (1, 3) ∉ R.

Answer: neither reflexive, nor symmetric, nor transitive

NCERT – Q4

Type: Check the three properties

Question: Show that R in R, R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

Solution:

Step 1: a ≤ a for every a: reflexive.

Step 2: 1 ≤ 2 but 2 ≤ 1 is false: not symmetric.

Step 3: a ≤ b and b ≤ c give a ≤ c: transitive.

Answer: reflexive, transitive, not symmetric

NCERT – Q5

Type: Counter-examples

Question: Check whether R in R, R = {(a, b) : a ≤ b^3}, is reflexive, symmetric or transitive.

Solution:

Step 1: Reflexive fails for a = ½:

$$\frac{1}{2} \leq \frac{1}{8} \text{ false}$$

Step 2: Symmetric fails for (1, 2):

$$1 \leq 8 \text{ true}, \quad 2 \leq 1 \text{ false}$$

Step 3: Transitive fails for (3, 3/2) and (3/2, 4/3):

$$3 \leq \frac{27}{8}, \quad \frac{3}{2} \leq \frac{64}{27}, \quad \text{but} \quad 3 \leq \frac{64}{27} \text{ false}$$

Answer: neither reflexive, nor symmetric, nor transitive

NCERT – Q6

Type: Check the three properties

Question: Show that R = {(1, 2), (2, 1)} on {1, 2, 3} is symmetric but neither reflexive nor transitive.

Solution:

Step 1: (1, 1) $\notin$ R: not reflexive. Both (1, 2) and (2, 1) are in R: symmetric.

Step 2: (1, 2), (2, 1) $\in$ R but (1, 1) $\notin$ R: not transitive.

Answer: symmetric only

NCERT – Q7

Type: Prove equivalence

Question: Show that R in the set A of all books in a library, R = {(x, y) : x and y have the same number of pages}, is an equivalence relation.

Solution:

Step 1: A book has the same pages as itself; “same pages” works both ways; if x, y and y, z have equal pages, so do x, z.

Answer: equivalence relation

NCERT – Q8

★ IMPORTANT · **Type:** Prove equivalence + classes

Question: Show that R in A = {1, 2, 3, 4, 5}, R = {(a, b) : |a − b| is even}, is an equivalence relation. Show that all elements of {1, 3, 5} are related to each other, all elements of {2, 4} are related to each other, and no element of {1, 3, 5} is related to any of {2, 4}.

Solution:

Step 1: Reflexive: |a − a| = 0 is even. Symmetric: |a − b| = |b − a|.

Step 2: Transitive: a − b and b − c even ⇒ a − c = (a − b) + (b − c) is even.

Step 3: Within {1, 3, 5} and within {2, 4} differences are even; an odd minus an even number is odd.

$$|1-2|=1,\ |3-4|=1,\ \ldots \text{ odd}$$

Answer: equivalence; classes {1, 3, 5} and {2, 4}

NCERT – Q9

★ IMPORTANT · **Type:** Prove equivalence + classes

Question: A = {x ∈ Z : 0 ≤ x ≤ 12}. Show that (i) R = {(a, b) : |a − b| is a multiple of 4}, (ii) R = {(a, b) : a = b} are equivalence relations, and find the set of all elements related to 1 in each case.

Solution:

Step 1: (i) |a − a| = 0 = 4 × 0; |a − b| = |b − a|; a − b = 4k, b − c = 4m ⇒ a − c = 4(k + m).

Step 2: Elements related to 1 differ from 1 by a multiple of 4:

$$\{1, 5, 9\}$$

Step 3: (ii) a = a; a = b ⇒ b = a; a = b, b = c ⇒ a = c. Only 1 is equal to 1:

$\{1\}$

Answer: (i) {1, 5, 9}; (ii) {1}

NCERT – Q10

Type: Make your own example

Question: Give an example of a relation which is (i) symmetric but neither reflexive nor transitive, (ii) transitive but neither reflexive nor symmetric, (iii) reflexive and symmetric but not transitive, (iv) reflexive and transitive but not symmetric, (v) symmetric and transitive but not reflexive.

Solution:

Step 1: Use A = {1, 2, 3}:

Step 2: (i) {(1, 2), (2, 1)}

Step 3: (ii) "<" on R: a < a is false, 1 < 2 but not 2 < 1, and < passes along.

Step 4: (iii) {(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)} – (1, 2), (2, 3) in but (1, 3) out.

Step 5: (iv) "≤" on R.

Step 6: (v) {(1, 1), (1, 2), (2, 1), (2, 2)} on A – (3, 3) is missing.

Answer: examples above (many answers are possible)

NCERT – Q11

Type: Prove equivalence

Question: Show that the relation R in the set A of points in a plane, R = {(P, Q) : distance of P from the origin = distance of Q from the origin}, is an equivalence relation. Show that the set of all points related to P ≠ (0, 0) is the circle through P with centre at the origin.

Solution:

Step 1: Equal distance: OP = OP; OP = OQ ⇒ OQ = OP; OP = OQ, OQ = OS ⇒ OP = OS.

Step 2: Points related to P are at distance OP from O:

$$\{Q:\ OQ = OP\} \ =\ \text{circle},\ \text{centre}\ O,\ \text{radius}\ OP$$

Answer: equivalence; class of P = circle with centre O through P

NCERT – Q12

★ **IMPORTANT** · **Type:** Prove equivalence

Question: Show that R in the set of all triangles, R = {(T_1, T_2) : T_1 is similar to T_2}, is an equivalence relation. Triangles T_1, T_2, T_3 have sides 3, 4, 5; 5, 12, 13 and 6, 8, 10. Which triangles are related?

Solution:

Step 1: Every triangle is similar to itself; similarity works both ways and passes along.

Step 2: Compare side ratios:

$$\frac{6}{3} = \frac{8}{4} = \frac{10}{5} = 2, \qquad \frac{5}{3} \neq \frac{12}{4}$$

Answer: equivalence; T_1 and T_3 are related

NCERT – Q13

Type: Prove equivalence

Question: Show that R in the set A of all polygons, R = {(P_1, P_2) : P_1 and P_2 have the same number of sides}, is an equivalence relation. What is the set of all elements related to the right-angled triangle with sides 3, 4, 5?

Solution:

Step 1: "Same number of sides" is reflexive, symmetric and transitive.

Step 2: The triangle has 3 sides, so it is related to every polygon with 3 sides.

Answer: equivalence; the set of all triangles

NCERT – Q14

Type: Prove equivalence

Question: Let L be the set of all lines in the XY-plane, R = {(L_1, L_2) : L_1 is parallel to L_2}. Show R is an equivalence relation. Find the set of all lines related to y = 2x + 4.

Solution:

Step 1: A line is parallel to itself (taken as parallel); parallel works both ways; $L_1 \parallel L_2$, $L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3$.

Step 2: Parallel lines have the same slope 2:

$$y = 2x + c, \quad c \in \mathbf{R}$$

Answer: equivalence; all lines y = 2x + c

NCERT – Q15

Type: MCQ

Question: Let R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)} on {1, 2, 3, 4}. Choose: (A) reflexive and symmetric but not transitive (B) reflexive and transitive but not symmetric (C) symmetric and transitive but not reflexive (D) an equivalence relation.

Solution:

Step 1: (1,1), (2,2), (3,3), (4,4) are all in R: reflexive.

Step 2: (1, 2) ∈ R but (2, 1) ∉ R: not symmetric.

Step 3: Chains: (1, 3), (3, 2) ⇒ (1, 2) ∈ R □; every other chain also closes: transitive.

Answer: (B)

NCERT – Q16

Type: MCQ

Question: Let R in N be R = {(a, b) : a = b − 2, b > 6}. Choose: (A) (2, 4) ∈ R (B) (3, 8) ∈ R (C) (6, 8) ∈ R (D) (8, 7) ∈ R.

Solution:

Step 1: Need b > 6 and a = b − 2. Check (6, 8): 8 > 6 and 6 = 8 − 2.

Answer: (C)

Exercise 1.2 · Types of Functions

One-one (injective)

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

✎ **Note:** To prove one-one: start from $f(x_1) = f(x_2)$ and reach $x_1 = x_2$. To disprove: find $x_1 \neq x_2$ with the same image.

Hinglish mein samjho:
$f(x_1) = f(x_2)$ se shuru karo aur **$x_1 = x_2$** tak pahuncho.

Onto (surjective)

$$y = f(x) \Rightarrow x = \ldots \in A$$

✎ **Note:** To prove onto: take any y in the codomain, solve $y = f(x)$ for x, and check that x is in the domain. To disprove: find one y that is never reached.

Hinglish mein samjho:
y = f(x) likh kar **x nikalo**, phir check karo ki x domain mein hai ya nahi.

Bijective

$$\text{bijective} = \text{one}-\text{one} + \text{onto}$$

✎ **Note:** Both one-one and onto = bijective. For a function from a finite set to itself, one-one and onto always come together.

Hinglish mein samjho:
One-one + onto. Finite set se khud tak – ek mila to doosra apne aap.

Hinglish mein samjho:
One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Onto: y se x nikalo aur domain check karo. Ek bhi fail to **bijective nahi**.

SOLVED EXAMPLE – Prove one-one and onto (Ex 1.2 type)

Step 1: f : R → R, f(x) = 3 − 4x. One-one: suppose $f(x_1) = f(x_2)$.

$$3 - 4x_1 = 3 - 4x_2 \Rightarrow x_1 = x_2$$

Step 2: Onto: take any real y and solve y = 3 − 4x.

$$x = \frac{3-y}{4} \in \mathbf{R}, \qquad f\left(\frac{3-y}{4}\right) = y$$

Step 3: Both hold:

$$f \text{ is bijective}$$

SOLVED EXAMPLE – Not one-one, not onto (Ex 1.2 type)

Step 1: f : R → R, f(x) = x². Two inputs, same image:

$$f(-2) = f(2) = 4$$

Why: −2 ≠ 2, so not one-one.

Step 2: A negative number is never a square:

$$f(x) = -1 \text{ has no real solution}$$

Why: so not onto.

Exam tip: For onto, always **write x in terms of y and check it lies in the domain** (for N, the answer must be a natural number).

Exercise 1.2 · All NCERT Questions Solved

12 questions. Board favourites: **prove bijective (Q7 (i), Q8, Q10)**, **x^2, x^3 on N, Z, R (Q2)** and the special functions (Q3–Q5).

NCERT – Q1

Type: One-one and onto

Question: Show that f : R* → R*, f(x) = 1/x is one-one and onto (R* = non-zero reals). Is the result true if the domain R* is replaced by N with the codomain still R*?

Solution:

Step 1: One-one:

$$\frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$$

Step 2: Onto: for any y ≠ 0, x = 1/y is non-zero and f(1/y) = y.

$$y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \in \mathbf{R}_*$$

Step 3: Domain N: still one-one, but y = 2 needs x = ½, which is not a natural number.

$$\text{not onto}$$

Answer: R* → R*: bijective; N → R*: one-one but not onto

NCERT – Q2 (i)

★ IMPORTANT · **Type:** Injective / surjective

Question: f : N → N, f(x) = x².

Solution:

Step 1: $x_1^2 = x_2^2 \Rightarrow x_1 = x_2$ (natural numbers are positive): one-one.

Step 2: 2 is not the square of any natural number: not onto.

Answer: one-one, not onto

NCERT – Q2 (ii)

Type: Injective / surjective

Question: f : Z → Z, f(x) = x².

Solution:

Step 1: f(−1) = f(1) = 1: not one-one. −1 has no pre-image: not onto.

Answer: neither one-one nor onto

NCERT – Q2 (iii)

Type: Injective / surjective

Question: f : R → R, f(x) = x².

Solution:

Step 1: f(−1) = f(1): not one-one. f(x) = −1 has no real solution: not onto.

Answer: neither one-one nor onto

NCERT – Q2 (iv)

Type: Injective / surjective

Question: $f : N \to N$, $f(x) = x^3$.

Solution:

Step 1: $x_1^3 = x_2^3 \Rightarrow x_1 = x_2$: one-one. 2 is not a cube of a natural number: not onto.

Answer: one-one, not onto

NCERT – Q2 (v)

Type: Injective / surjective

Question: $f : Z \to Z$, $f(x) = x^3$.

Solution:

Step 1: $x_1^3 = x_2^3 \Rightarrow x_1 = x_2$: one-one. 2 is not the cube of an integer: not onto.

Answer: one-one, not onto

NCERT – Q3

★ **IMPORTANT** · **Type:** Special functions

Question: Prove that the Greatest Integer Function $f : R \to R$, $f(x) = [x]$, is neither one-one nor onto.

Solution:

Step 1: $f(1.2) = f(1.9) = 1$ although $1.2 \neq 1.9$: not one-one.

Step 2: Outputs are only integers, so 0.7 has no pre-image: not onto.

Answer: neither one-one nor onto

NCERT – Q4

Type: Special functions

Question: Show that the Modulus Function f : R → R, f(x) = |x|, is neither one-one nor onto.

Solution:

Step 1: |−1| = |1| = 1: not one-one. |x| is never negative, so −1 has no pre-image: not onto.

Answer: neither one-one nor onto

NCERT – Q5

Type: Special functions

Question: Show that the Signum Function f : R → R (1 if x > 0, 0 if x = 0, −1 if x < 0) is neither one-one nor onto.

Solution:

Step 1: f(1) = f(2) = 1: not one-one. Outputs are only −1, 0, 1, so 2 has no pre-image: not onto.

Answer: neither one-one nor onto

NCERT – Q6

Type: Function as a set

Question: A = {1, 2, 3}, B = {4, 5, 6, 7}, f = {(1, 4), (2, 5), (3, 6)}. Show that f is one-one.

Solution:

Step 1: Different elements of A have different images 4, 5, 6.

Answer: one-one (not onto, since 7 has no pre-image)

NCERT – Q7 (i)

★ IMPORTANT · **Type:** Prove bijective

Question: f : R → R, f(x) = 3 − 4x. Is f one-one, onto or bijective?

Solution:

Step 1: $3 - 4x_1 = 3 - 4x_2 \Rightarrow x_1 = x_2$: one-one.

Step 2: Onto: x = (3 − y)/4 is real for every real y.

$$f\left(\frac{3-y}{4}\right) = 3 - (3 - y) = y$$

Answer: bijective

NCERT – Q7 (ii)

Type: Injective / surjective

Question: f : R → R, f(x) = 1 + x².

Solution:

Step 1: f(−1) = f(1) = 2: not one-one. 1 + x² ≥ 1, so 0 has no pre-image: not onto.

Answer: neither one-one nor onto

NCERT – Q8

Type: Prove bijective

Question: Let A and B be sets. Show that f : A × B → B × A, f(a, b) = (b, a), is bijective.

Solution:

Step 1: One-one: $(b_1, a_1) = (b_2, a_2) \Rightarrow b_1 = b_2, a_1 = a_2 \Rightarrow (a_1, b_1) = (a_2, b_2)$.

Step 2: Onto: any $(b, a) \in B \times A$ is f(a, b).

Answer: bijective

NCERT – Q9

★ IMPORTANT · **Type:** Piecewise function

Question: f : N → N, f(n) = (n + 1)/2 if n is odd, n/2 if n is even. Is f bijective?

Solution:

Step 1: f(1) = 1 and f(2) = 1: not one-one.

Step 2: For any natural n, f(2n) = n: onto.

Answer: onto but not one-one, so not bijective

NCERT – Q10

★ IMPORTANT · **Type:** Prove bijective

Question: A = R − {3}, B = R − {1}, f : A → B, f(x) = (x − 2)/(x − 3). Is f one-one and onto?

Solution:

Step 1: One-one: cross-multiply $f(x_1) = f(x_2)$.

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3) \Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$$

Step 2: Onto: solve y = (x − 2)/(x − 3) for x.

$$y(x - 3) = x - 2 \Rightarrow x = \frac{3y - 2}{y - 1}$$

Step 3: This x exists because y ≠ 1, and x ≠ 3 (else 3y − 3 = 3y − 2).

$$f\left(\frac{3y - 2}{y - 1}\right) = y$$

Answer: one-one and onto (bijective)

NCERT – Q11

Type: MCQ

Question: f : R → R, f(x) = x^4. Choose: (A) one-one onto (B) many-one onto (C) one-one but not onto (D) neither one-one nor onto.

Solution:

Step 1: f(−1) = f(1): many-one. −1 is not a fourth power: not onto.

Answer: (D)

NCERT – Q12

Type: MCQ

Question: f : R → R, f(x) = 3x. Choose: (A) one-one onto (B) many-one onto (C) one-one but not onto (D) neither.

Solution:

Step 1: $3x_1 = 3x_2 \Rightarrow x_1 = x_2$; any y = 3(y/3).

Answer: (A)

Miscellaneous Exercise · Mixed Questions

Hinglish mein samjho:
Misc mein mixed sawal: modulus wale function (case x ≥ 0, x < 0 alag), subsets wala relation, aur counting (n!).

NCERT – Q1

★ **IMPORTANT** · **Type:** Prove bijective

Question: Show that f : R → {x ∈ R : −1 < x < 1}, f(x) = x/(1 + |x|), is one-one and onto.

Solution:

Step 1: For x ≥ 0, f(x) = x/(1 + x) lies in [0, 1); for x < 0, f(x) = x/(1 − x) lies in (−1, 0). So f(x₁) = f(x₂) forces x₁, x₂ to have the same sign.

Step 2: Same sign, x ≥ 0:

$$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1 = x_2$$

Step 3: Onto: for 0 ≤ y < 1 take x = y/(1 − y) ≥ 0; for −1 < y < 0 take x = y/(1 + y) < 0.

$$f\left(\frac{y}{1-y}\right) = y$$

Answer: one-one and onto

NCERT – Q2

Type: One-one

Question: Show that f : R → R, f(x) = x³, is injective.

Solution:

Step 1: $x_1^3 = x_2^3 \Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$; the second bracket is 0 only when x₁ = x₂ = 0.

$$x_1 = x_2$$

Answer: injective (one-one)

NCERT – Q3

Type: Relation on subsets

Question: Given a non-empty set X, consider P(X), the set of all subsets of X. Define R on P(X): A R B iff A ⊂ B. Is R an equivalence relation?

Solution:

Step 1: A ⊂ A: reflexive. A ⊂ B, B ⊂ C ⇒ A ⊂ C: transitive.

Step 2: φ ⊂ X but X ⊄ φ: not symmetric.

Answer: not an equivalence relation (not symmetric)

NCERT – Q4

Type: Counting

Question: Find the number of all onto functions from {1, 2, 3, …, n} to itself.

Solution:

Step 1: From a finite set to itself, onto = one-one = a rearrangement (permutation).

$n!$

Answer: n!

NCERT – Q5

Type: Equal functions

Question: A = {−1, 0, 1, 2}, B = {−4, −2, 0, 2}, f(x) = x² − x and g(x) = 2|x − ½| − 1. Are f and g equal?

Solution:

Step 1: Compare values at each point of A:

$$f(-1)=2=g(-1),\quad f(0)=0=g(0),\quad f(1)=0=g(1),\quad f(2)=2=g(2)$$

Answer: yes, f = g (same domain, codomain and values)

NCERT – Q6

Type: MCQ

Question: A = {1, 2, 3}. The number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is (A) 1 (B) 2 (C) 3 (D) 4.

Solution:

Step 1: Must contain (1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1). This is not transitive: (2,1), (1,3) but (2,3) missing.

Step 2: Adding (2, 3) forces (3, 2) and makes it transitive (the universal relation).

Answer: (A) 1

NCERT – Q7

Type: MCQ

Question: A = {1, 2, 3}. The number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4.

Solution:

Step 1: Smallest one: {(1,1), (2,2), (3,3), (1,2), (2,1)}. The only bigger one is A × A.

Answer: (B) 2

Important Questions and Question Types · Relations and Functions

Question type	Steps	Marks
Prove equivalence relation	Check reflexive, symmetric, transitive with general a, b, c; write the equivalence class if asked.	3–5
Not reflexive / symmetric / transitive	One counter-example with actual numbers.	1–2
Prove one-one	$f(x_1) = f(x_2) \Rightarrow \ldots \Rightarrow x_1 = x_2$.	2
Prove onto	$y = f(x) \Rightarrow x = \ldots$; check x is in the domain.	2
Not one-one / not onto	Two inputs with the same image / one y with no pre-image.	1–2
Counting	relations 2^{n^2}; functions n^m; onto from a set to itself n!	1

Question Bank to Memorise

1-mark questions

Q1. How many relations are there on a set with 2 elements? [1]

Ans. $2^{2^2} = 16$.

Q2. Is the empty relation on A = {1, 2} reflexive? [1]

Ans. No – (1, 1) is missing.

Q3. Is the relation "is a sibling of" reflexive? [1]

Ans. No – nobody is their own sibling.

Q4. f : N → N, f(x) = 2x. Is f onto? [1]

Ans. No – odd numbers have no pre-image.

Q5. Number of bijections from {1, 2, 3} to itself? [1]

Ans. 3! = 6.

Q6. Equivalence class of 0 for R = {(a, b) : a − b divisible by 2} on Z? [1]

Ans. All even integers.

Assertion–Reason

A: both true, R explains A. B: both true, R doesn't explain A. C: A true, R false. D: A false, R true.

QAR1. A: f : R → R, f(x) = x² is not one-one. R: f(−2) = f(2). [1]

Ans. A

QAR2. A: The relation R = {(a, b) : a ≤ b} on R is an equivalence relation. R: It is reflexive and transitive. [1]

Ans. D – A is false (not symmetric); R is true.

QAR3. A: Every onto function from a finite set to itself is one-one. R: For a finite set, onto and one-one go together. [1]

Ans. A

3-mark and 5-mark questions

QL1. Show that R = {(a, b) : 2 divides a − b} on Z is an equivalence relation and find [0] and [1]. [3]

Ans. Reflexive (0), symmetric (−(a − b)), transitive (sum). [0] = even integers, [1] = odd integers.

QL2. Show that f : R − {−4/3} → R − {4/3}, f(x) = 4x/(3x + 4) is one-one and onto. [3]

Ans. $f(x_1) = f(x_2) \Rightarrow 16x_1 = 16x_2$. Onto: x = 4y/(4 − 3y), defined as y ≠ 4/3.

QL3. Show that R = {(a, b) : |a − b| is a multiple of 4} on {0, 1, …, 12} is an equivalence relation and write all classes. [5]

Ans. As NCERT Q9. Classes {0, 4, 8, 12}, {1, 5, 9}, {2, 6, 10}, {3, 7, 11}.

Last-Minute Checklist

- I can test reflexive, symmetric and transitive with general elements.
- I give a counter-example with actual numbers when a property fails.
- I can prove an equivalence relation and write its classes.
- I can prove one-one using $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
- I can prove onto by solving $y = f(x)$ and checking the domain.
- I know $[x]$, $|x|$ and sgn x are neither one-one nor onto on R.
- I know the counts: 2^{n^2} relations, n! bijections.